

Marcus Neuer

Finite Larmor radius contributions to anomalous transport in plasmas with stochastic magnetic fields

Physikalische Motivation

Analytisches Verfahren für das A-Langevin Modell

Diffusionsregime

Numerische Simulationen

Zusammenfassung und Ausblick

Transport im Tokamak (u.a. Verunreinigungen)

$$\chi_{\perp,i} = \frac{v_t^2 \nu}{2 \Omega_L^2} \sim 10^{-4} \text{ m}^2 / \text{s}$$

$$D_{\perp} = \chi_{\perp} + D_{\perp}^{\text{AN}} \sim 1 \text{ m}^2 / \text{s}$$

Mattioli et al. (ToreSupra)

Galli et al. (JET)

TEXTOR-DED

Cosmic rays

$$\rho_{L,\text{cr}} \approx 1 \text{ AE}$$

$$\Delta \approx 10^7 \rho_{L,\text{cr}}$$

$$B \sim 1 \mu\text{G}$$

Jokipii et al.

Cassé et al.

Einfluß der **endlichen Larmor Radien**?

Die A-Langevin Gleichung

$$\mathbf{B} = B_0 (b_0 \mathbf{e}_z + \mathbf{b})$$

$$\dot{\mathbf{u}}(t) = \frac{Ze}{mc} \mathbf{u}(t) \times \mathbf{B}(t) - \nu \mathbf{u}(t) + \mathbf{a}(t)$$

- Maxwellverteilung der Anfangsgeschwindigkeiten
- Delta-Distribution als Korrelation der Stöße
- **Lagrangekorrelationsfunktion** des Störfelds

$$\text{Kubo-Zahl: } \kappa = \frac{\beta \lambda_{\parallel}}{b_0 \lambda_{\perp}}$$

Bestimmung der Transportgrößen aus der A-Langevin Gl.

Geschwindigkeit



**Geschwindigkeits-
korrelationsfunktion**



Lagrangekorrelationsfunktion

(1) Corrsin Approximation

(2) Decorrelation Trajectory Method



Green-Kubo-Formalismus

Differentialgleichung für MSD
(mean-square-displacement)

Differentialgleichung für die MSD

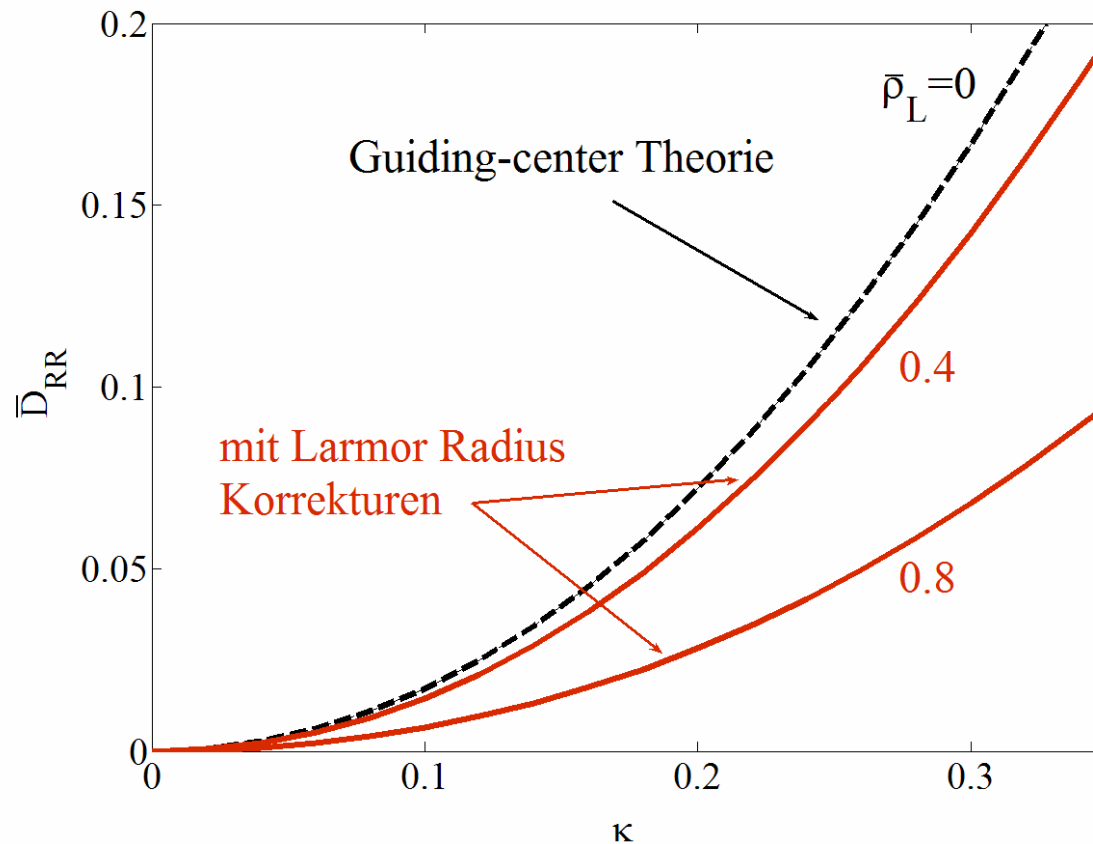
Störungstheorie für starke Führungsfelder

Beiträge der **Guiding-center** Bewegung

$$\frac{d^2 \langle \delta x^2 (t) \rangle}{dt^2} = 2 \frac{dD(t)}{dt} = K + L^{(0)} + L^{(1)}$$

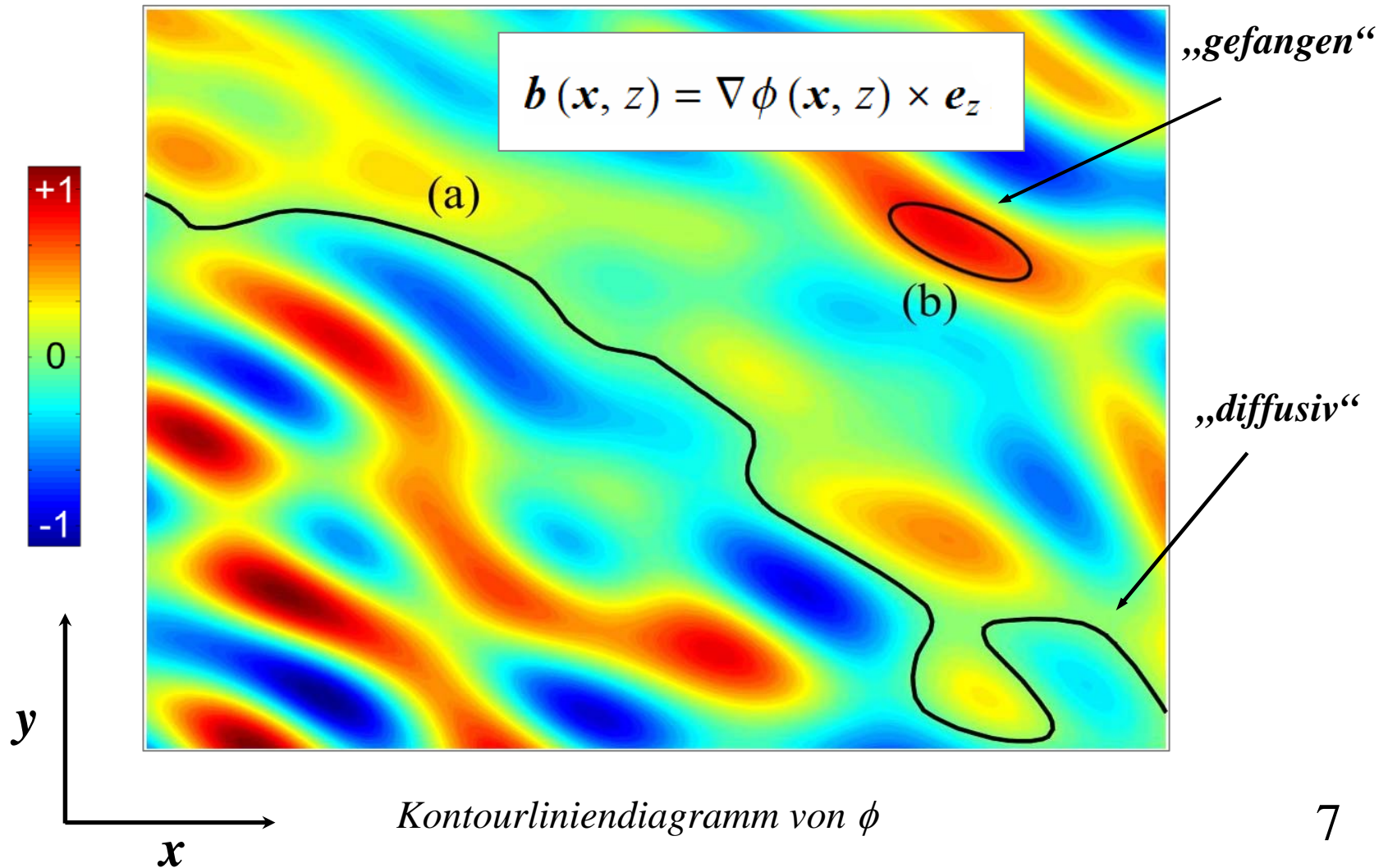
Beiträge der **Larmor-Radien**
(in erster Ordnung Störungstheorie)

Einfluss des Larmor Radius am Beispiel des Rechester-Rosenbluth-Regime (kleine Kubo-Zahlen)



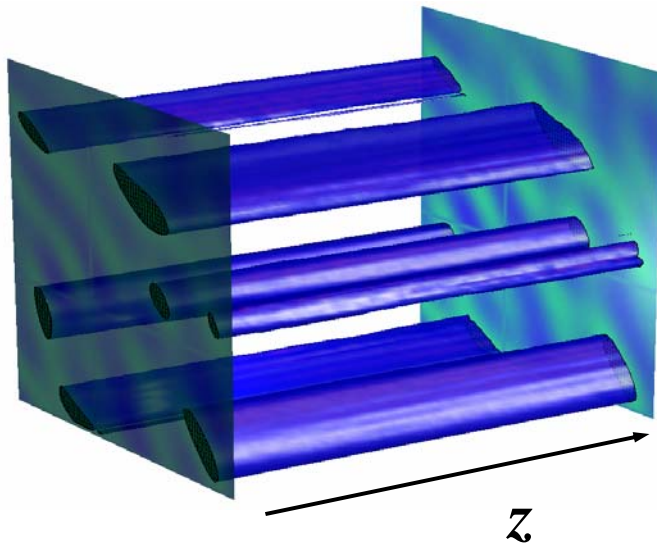
stoßbestimmt mit endlichen Korrelationslängen

Perkulationsregime (hohe Kubo-Zahlen)



Perkulationsregime

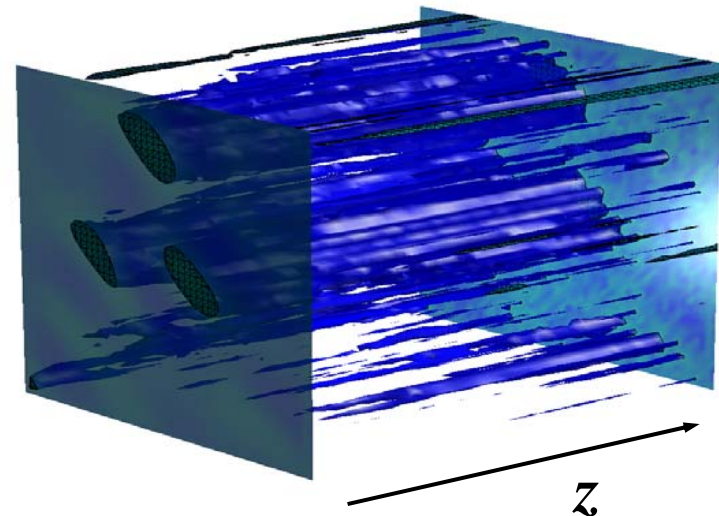
„hohe parallele Korrelation“



$$\kappa = \frac{\beta \lambda_{\parallel}}{b_0 \lambda_{\perp}} \geq 1. \quad \text{nur DCT}$$

„Perkulationsregime“

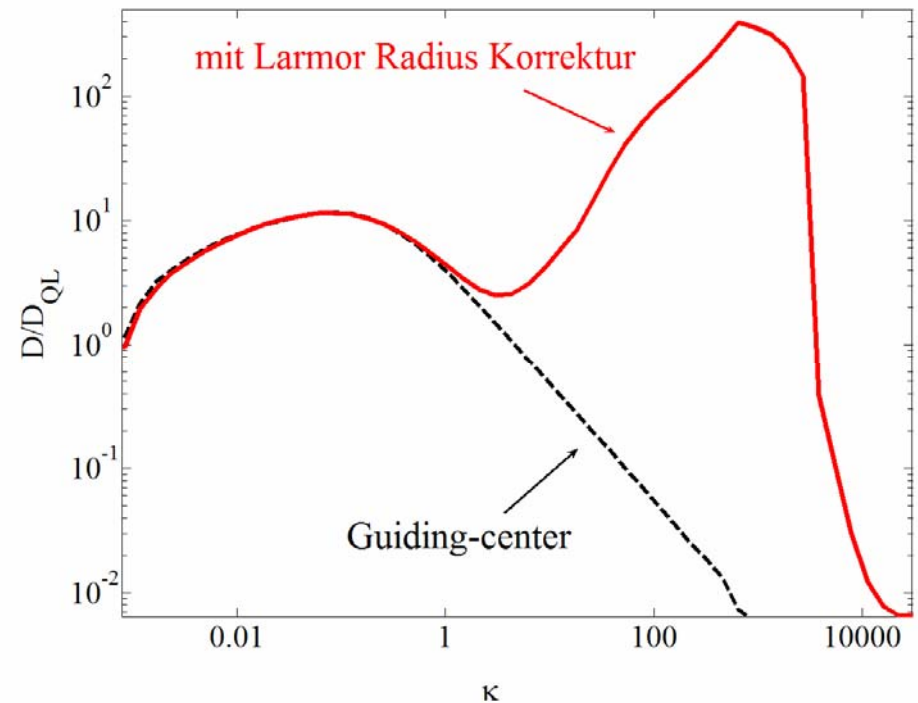
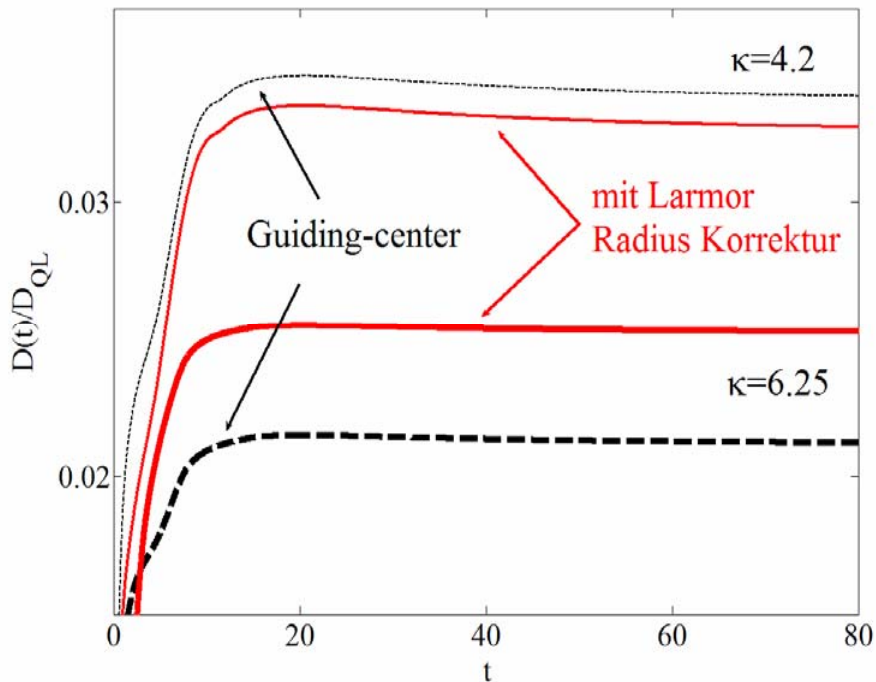
„geringe parallele Korrelation“



$$\kappa = \frac{\beta \lambda_{\parallel}}{b_0 \lambda_{\perp}} \ll 1. \quad \text{Corrsin \& DCT}$$

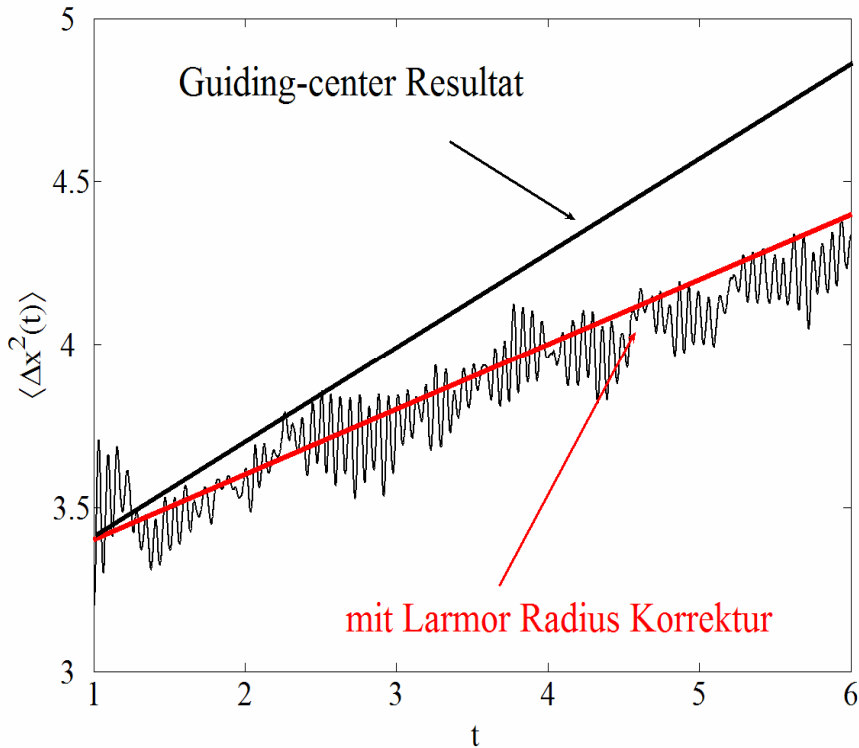
Perkulationsregime

Perkulationsstruktur \rightarrow Verringerung der Diffusion

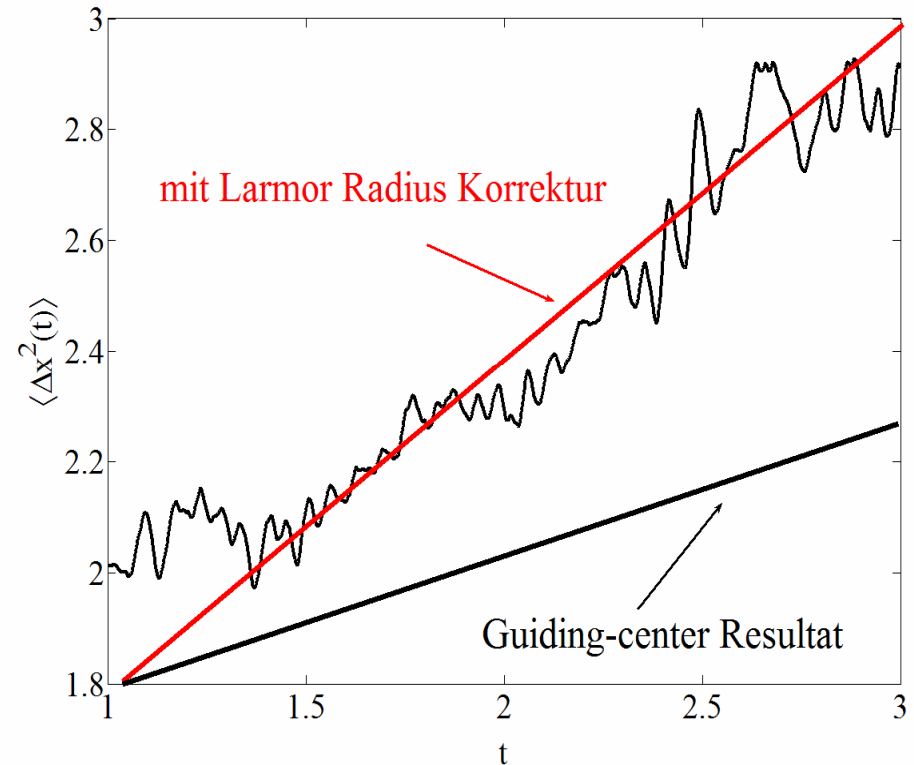


Larmor Radien \rightarrow Erhöhung der Diffusion

Ergebnisse für Corrsin-Approximation und DCT



**Rechester-Rosenbluth-Regime
(Corrsin)**



**Perkolationsregime
(DCT)**

Zusammenfassung

- **Neues analytisches Modell** auf der Basis der A-Langevin Gl.
- Berücksichtigung der Gyration → **finite Larmor Radius Effekte**
- Beispiele für **anomale** Diffusionsregime und ihre Korrekturen
- Bestätigung des Modells mit **numerischen Simulationen**

Ausblick

- Berücksichtigung von **stochastischen elektrischen Feldern**

Corrsin Approximation

$$\mathcal{E}(x, y, z) = \begin{pmatrix} 1 - \frac{y^2}{\lambda_{\perp}^2} & 0 \\ 0 & 1 - \frac{x^2}{\lambda_{\perp}^2} \end{pmatrix} \exp\left(-\frac{x^2 + y^2}{2\lambda_{\perp}^2} - \frac{z^2}{2\lambda_{\parallel}^2}\right)$$

$$\mathcal{L}(t) = \int d\mathbf{r} \mathcal{E}(\mathbf{r}) \langle \delta(\mathbf{r} - \mathbf{R}(t)) \rangle$$

$$\mathcal{L}_{\perp} = \frac{1}{\sqrt{1 + \delta z^2 / \lambda_{\parallel}^2}} \frac{1}{(1 + \delta x^2 / \lambda_{\parallel}^2)^2} \left\{ 1 - \frac{\varphi^2}{1 + \delta z^2 / \lambda_{\parallel}^2} \right\}$$

Decorrelation Trajectory Method

$$A(\mathbf{x}, z) = \langle \phi(\mathbf{0}, 0) \phi(\mathbf{x}, z) \rangle = \beta^2 \lambda_{\perp}^2 \exp\left(-\frac{x^2 + y^2}{2\lambda_{\perp}^2} - \frac{z^2}{2\lambda_{\parallel}^2}\right)$$

$$\mathcal{E} = \langle \mathbf{b}(0) \otimes \mathbf{b}(x) \rangle = \langle (\nabla \times \phi \mathbf{e}_z) \otimes (\nabla \times \phi \mathbf{e}_z) \rangle$$

$$\mathbf{F}(\mathbf{x}(t)) \equiv \langle \mathbf{b}(\mathbf{x}) \rangle_S = \begin{pmatrix} b_x^0 \mathcal{E}_{xx} + b_y^0 \mathcal{E}_{xy} + \phi^0 \mathcal{E}_{\phi x} \\ b_x^0 \mathcal{E}_{yx} + b_y^0 \mathcal{E}_{yy} + \phi^0 \mathcal{E}_{\phi y} \\ 0 \end{pmatrix}$$

Balescu, Vlad et.al. 2003

$$\beta^2 \mathcal{L}(t) = \int d\phi^0 d\mathbf{b}^0 P(\mathbf{b}^0) P(\phi^0) \mathbf{b}(\mathbf{0}) \otimes \mathbf{F}(\mathbf{X}(t), t)$$

DCT-Approximation

$\mathbf{X}(t)$ from simulations

Euler Vs. Lagrange

bisher: Korrelationsfunktionen in Eulerkoordinaten (Laborsystem)

$$\langle\langle\langle\mathbf{u}(\mathbf{r}_1, \tau_1) \mathbf{u}(\mathbf{r}_2, \tau_2)\rangle\rangle_a\rangle_{u_0}\rangle_b$$

benötigt: **Lagrangekorrelationsfunktionen** (mitbewegtes Bezugssystem)

$$\langle\langle\langle\mathbf{u}(t_1) \mathbf{u}(t_2)\rangle\rangle_a\rangle_{u_0}\rangle_b$$

mitbewegt = Kenntnis der (stochastischen) Teilchenbahn $\mathbf{R}(t)$.

Wahl der Näherungsmethode ist abhängig von der Größe der Kubo-Zahl:

$$\kappa = \frac{V\tau_c}{\lambda_{\perp}} = \frac{\beta\lambda_{\parallel}}{b_0\lambda_{\perp}}$$

(1) **Corrsin Approximation** $\kappa < 1$.

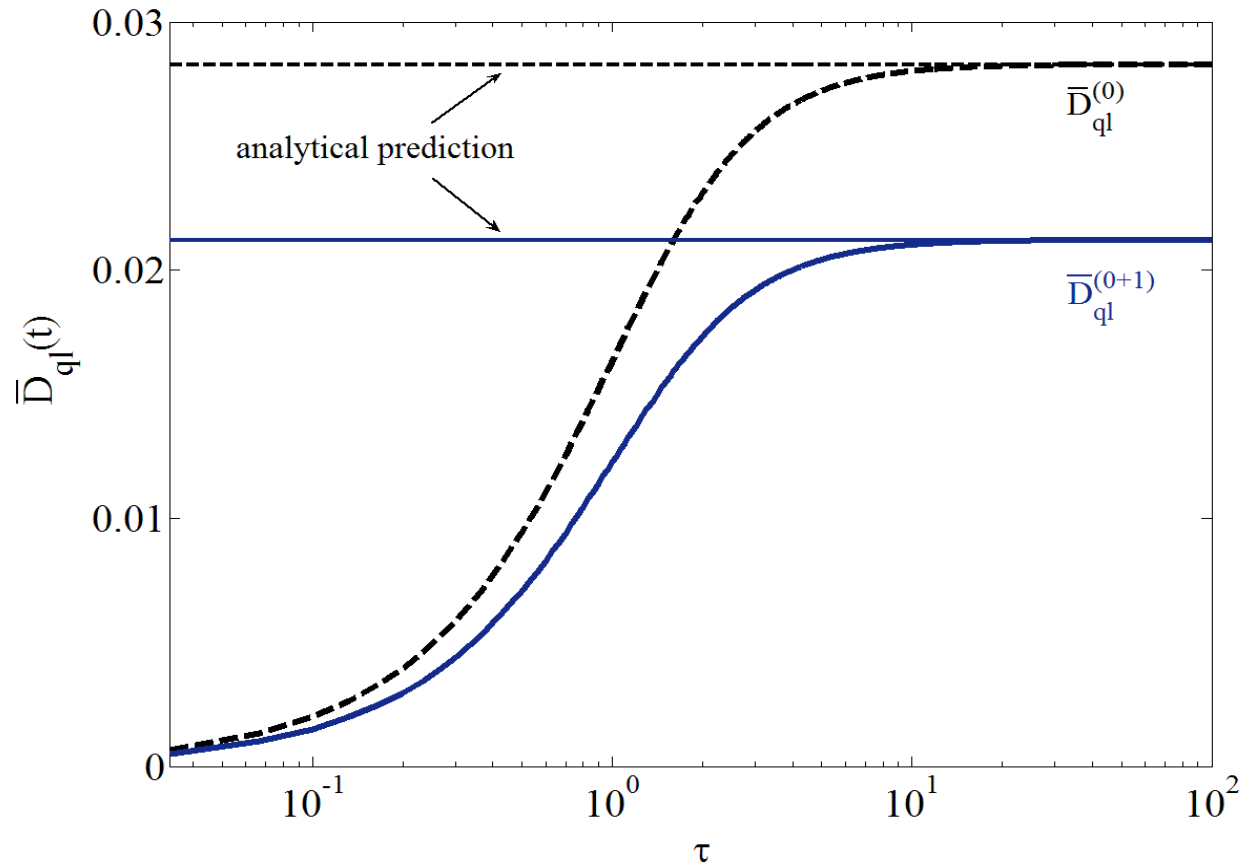
(2) **Decorrelation-Trajectory Methode (DCT)**

Vlad et al.

Differentialgleichung für die MSD

$$\begin{aligned}
 \frac{d^2}{dt^2} \langle \delta x^2(t) \rangle^{\text{AN}} = & \\
 2 \frac{d}{dt} D^{\text{AN}}(t) = & \frac{\beta^2}{b_0^2} \left[\frac{\chi_{\parallel} v e^{-\nu t}}{\left(1 + \frac{\psi_{\parallel}(\nu t)}{\lambda_{\parallel}^2}\right)^{1/2}} - \frac{\varphi_{\parallel}^2}{\lambda_{\parallel}^2} \frac{1}{\left(1 + \frac{\psi_{\parallel}(\nu t)}{\lambda_{\parallel}^2}\right)^{3/2}} \right] \frac{1}{\left(1 + \frac{\langle \delta x^2(t) \rangle}{\lambda_{\perp}^2}\right)^2} \\
 - \frac{\rho_L^2}{v_t^2 b_0^2} \mathcal{L}^{(0)}_{\lambda_{\perp} \rightarrow \infty} & \left[\frac{4 \chi_{\perp} v e^{-\nu t}}{\left(1 + \frac{\psi_{\perp}}{\lambda_{\perp}^2}\right)^3 \lambda_{\perp}^2} - \frac{\varphi_{\perp}^2}{\lambda_{\perp}^4} \frac{18}{\left(1 + \frac{\psi_{\perp}}{\lambda_{\perp}^2}\right)^4} \right] \\
 - \frac{\rho_L^2 \beta^2}{b_0^2} \frac{v_t^2}{4 \lambda_{\parallel}^2 \left(1 + \frac{\psi_{\parallel}}{\lambda_{\parallel}^2}\right)^{3/2} \left(1 + \frac{\psi_{\perp}}{\lambda_{\perp}^2}\right)^2} &
 \end{aligned}$$

Quasilineares Regime



Diffusion und MSD

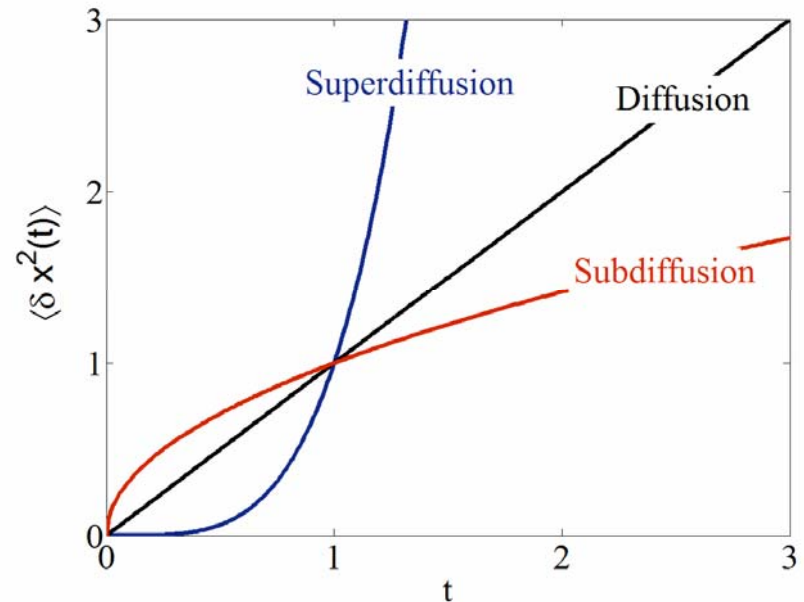
Klassifikation verschiedener Transportformen:

$$\langle \delta x^2(t) \rangle \sim t^\alpha$$

Definition des Diffusionskoeffizienten:

$$\langle \delta x^2(t) \rangle = 2 D(t) t$$

$$D(t) = \frac{1}{2} \frac{d}{dt} \langle \delta x^2(t) \rangle$$



Klassischer Transport:

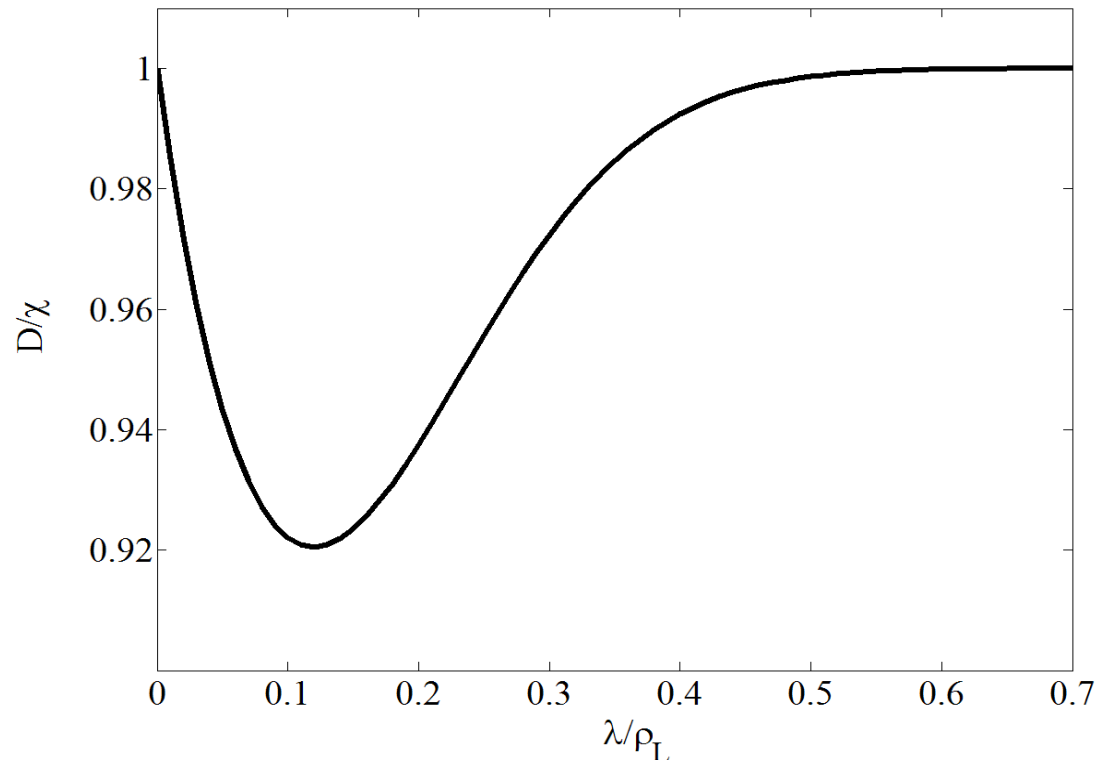
$$\langle \delta x^2(t) \rangle = \frac{v_t^2}{(v^2 + \Omega^2 b_0^2)} vt \equiv 2 \chi_\perp t$$

$$\langle \delta z^2(t) \rangle = \frac{v_t^2}{v} t \equiv 2 \chi_\parallel t$$

Diffusion bei verschwindenden Führungsfeldern

Stochastisches Störfeld =
Quelle zusätzlicher Stöße

$$\frac{D}{\chi} = \frac{1}{\left(1 + \frac{4\lambda}{\sqrt{2\pi}\rho_L} \operatorname{erfc}\left(\frac{\sqrt{2}\pi\lambda}{\rho_L}\right)\right)}$$



V-Langevin Gl.

$$\dot{\mathbf{u}}(t) = \frac{Ze}{mc} \mathbf{u}(t) \times \mathbf{B}(t) - \nu \mathbf{u}(t) + \mathbf{a}(t)$$

*Guiding center
approximation*

$$\frac{d\mathbf{r}_\perp}{dt} = \mathbf{b} \eta_\parallel + \boldsymbol{\eta}_\perp$$

*Integration in
der Zeit*

$$\frac{dz}{dt} = \eta_\parallel$$

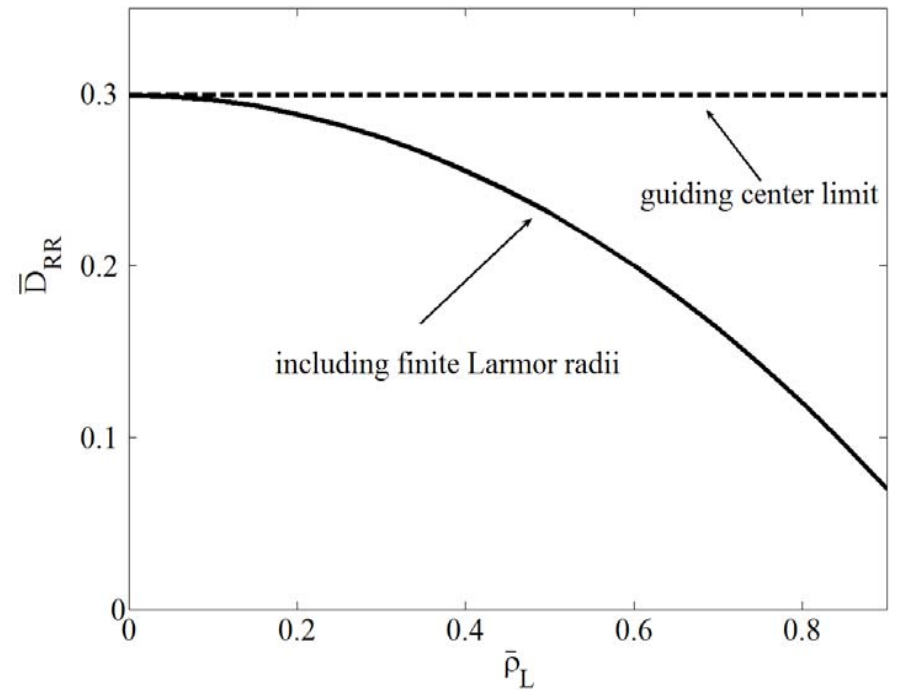
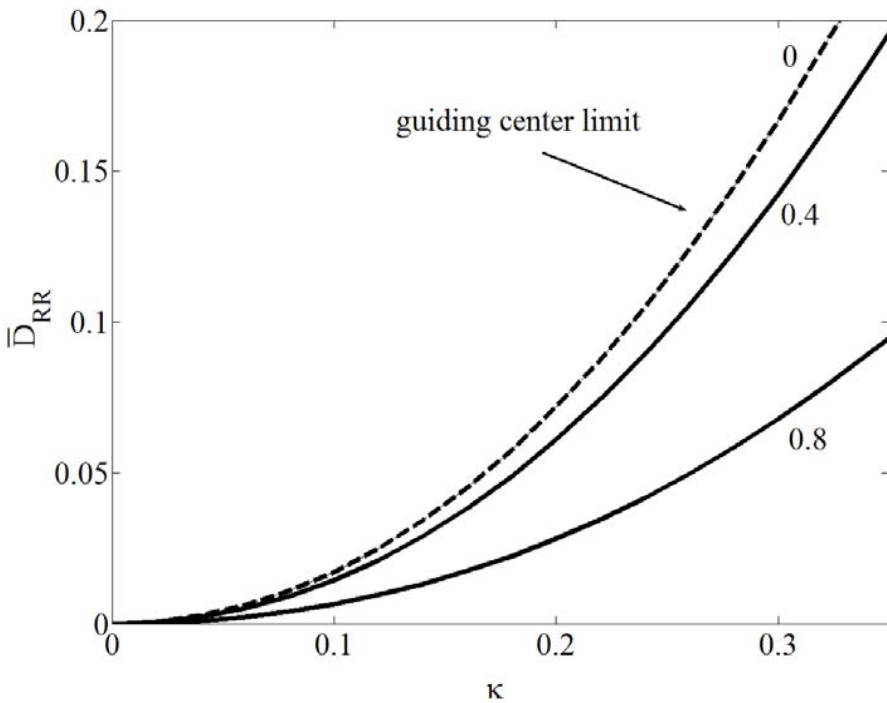
V-Langevin Gleichungen

Wang, Vlad, Vanden-Eijnden et al.

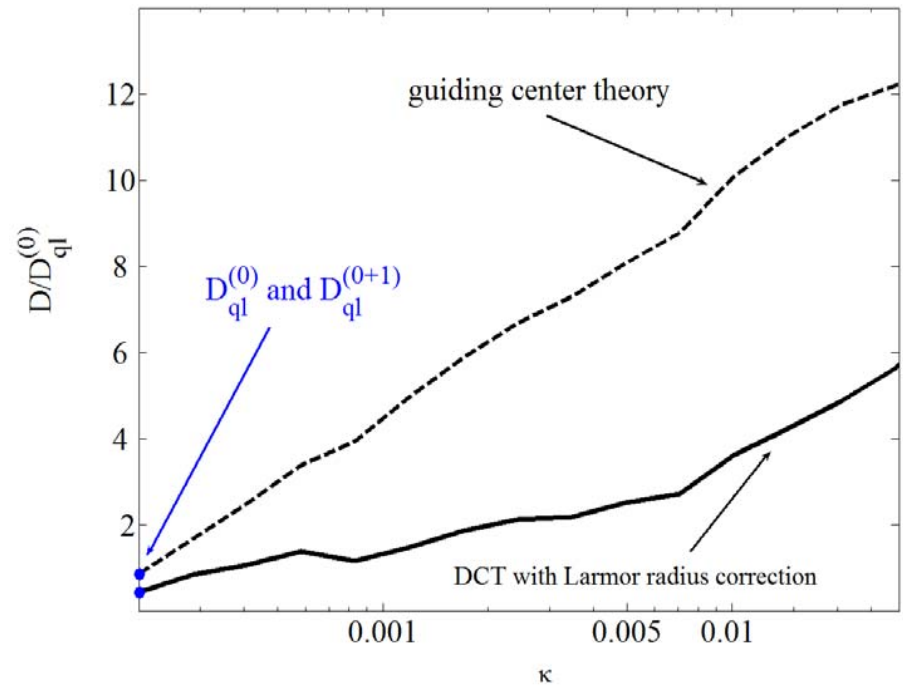
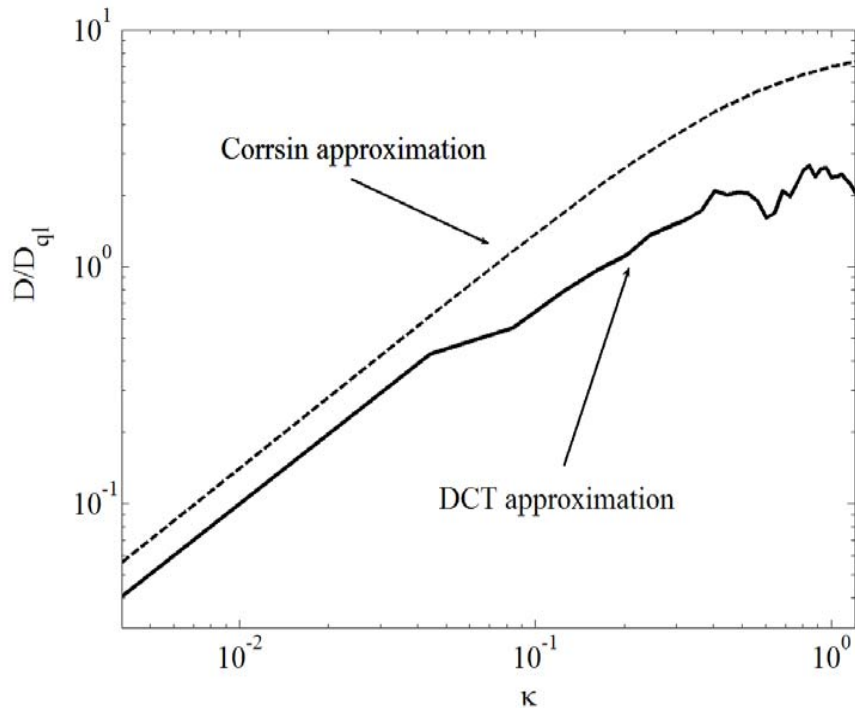
Rechester-Rosenbluth-Regime

$$\lambda_{\perp} \approx \lambda_{\parallel}$$

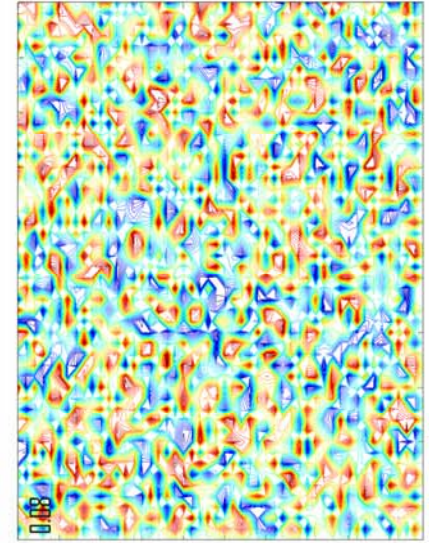
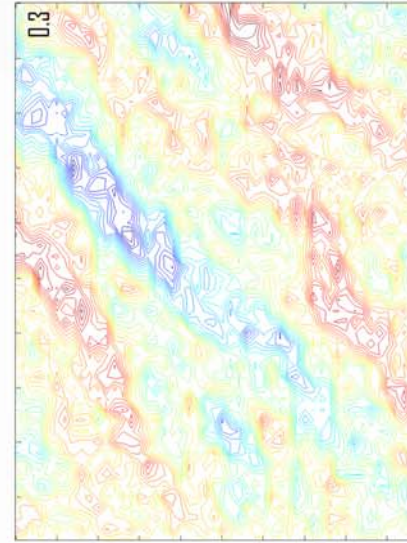
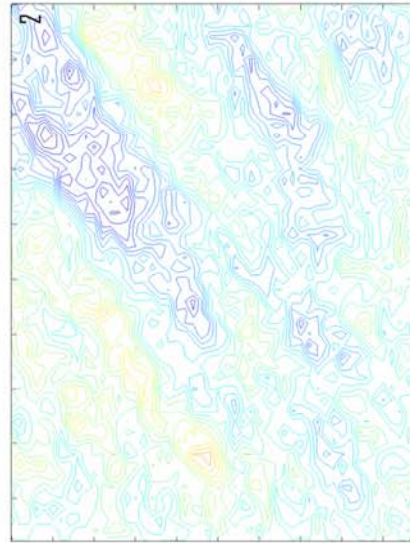
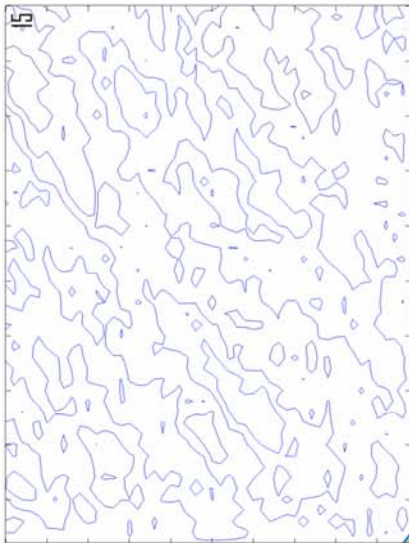
$$\bar{D}_{RR}^{(0)}(t) = \frac{16 \kappa^4 \bar{\chi}_{\parallel}}{\log^2(16 \sqrt{2} \pi \kappa^4 \bar{\chi}_{\parallel} \bar{\chi}_{\perp}^{-1})}$$



Corrsin Vs. DCT



Verschiedene Kontourdiagramme



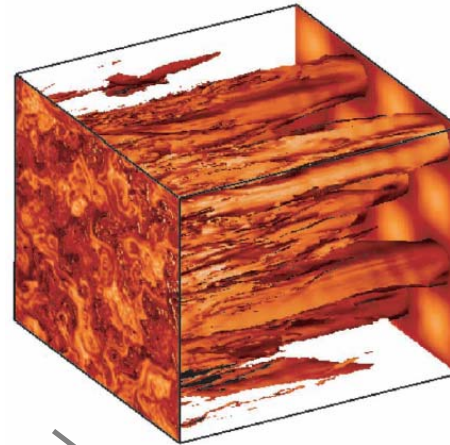
Reduktion von λ_{\perp}

Isichenko's Prognose

Diffusion skaliert mit
fraktaler Dimension ν

$$D \sim K^{-1/(\nu+2)}$$

$$D \sim K^{-0.7}$$

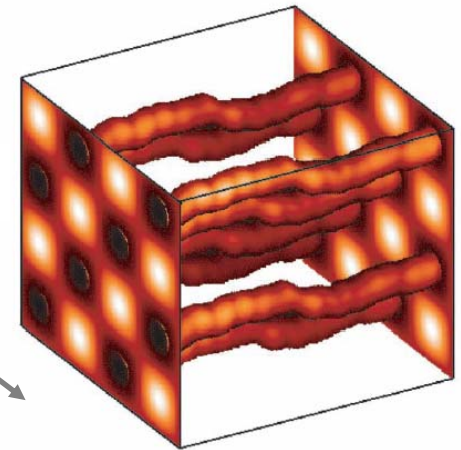


e.g. $\lambda_{||} \ll 1$

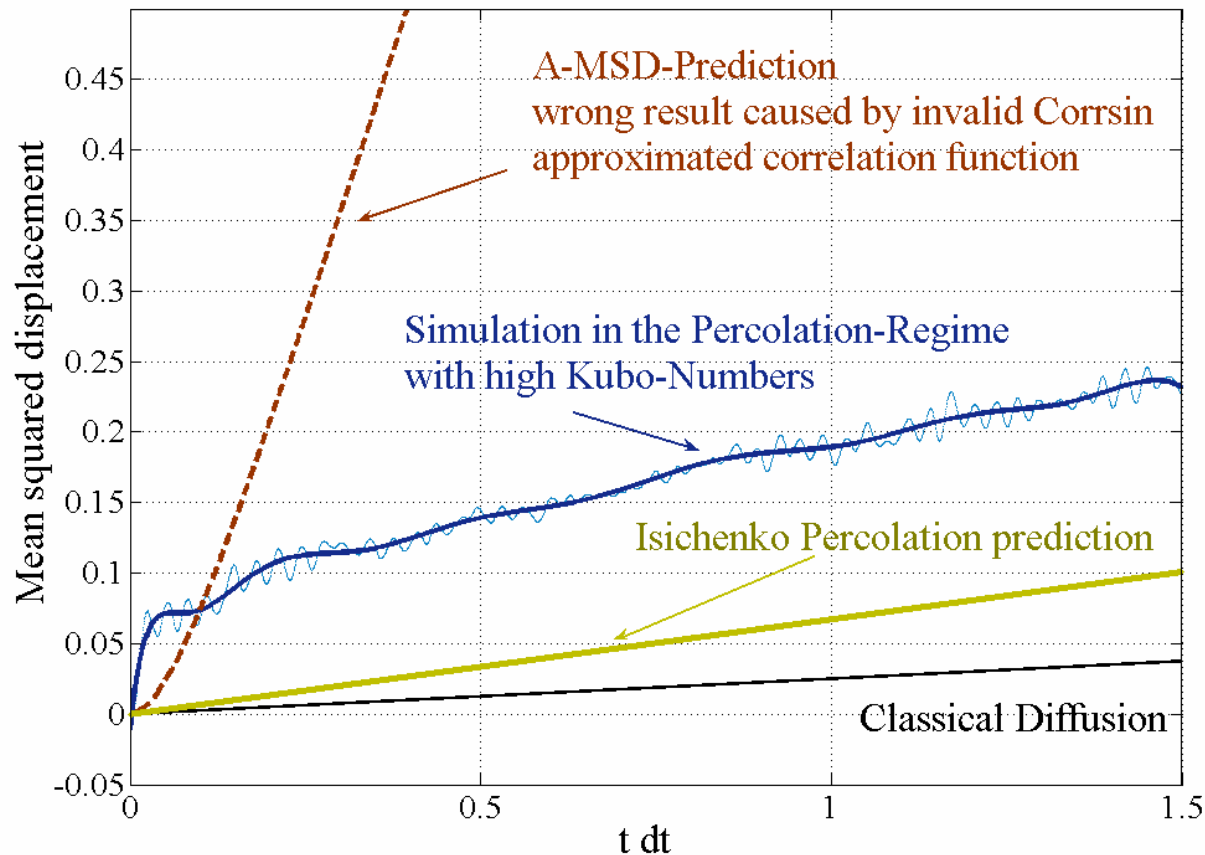
pictures by
Matthaeus et.al. 2003

e.g. $\lambda_{||} \gg 1$

Loss of Corrsins validity



Monte-Carlo-Simulation Isichenko Prognose

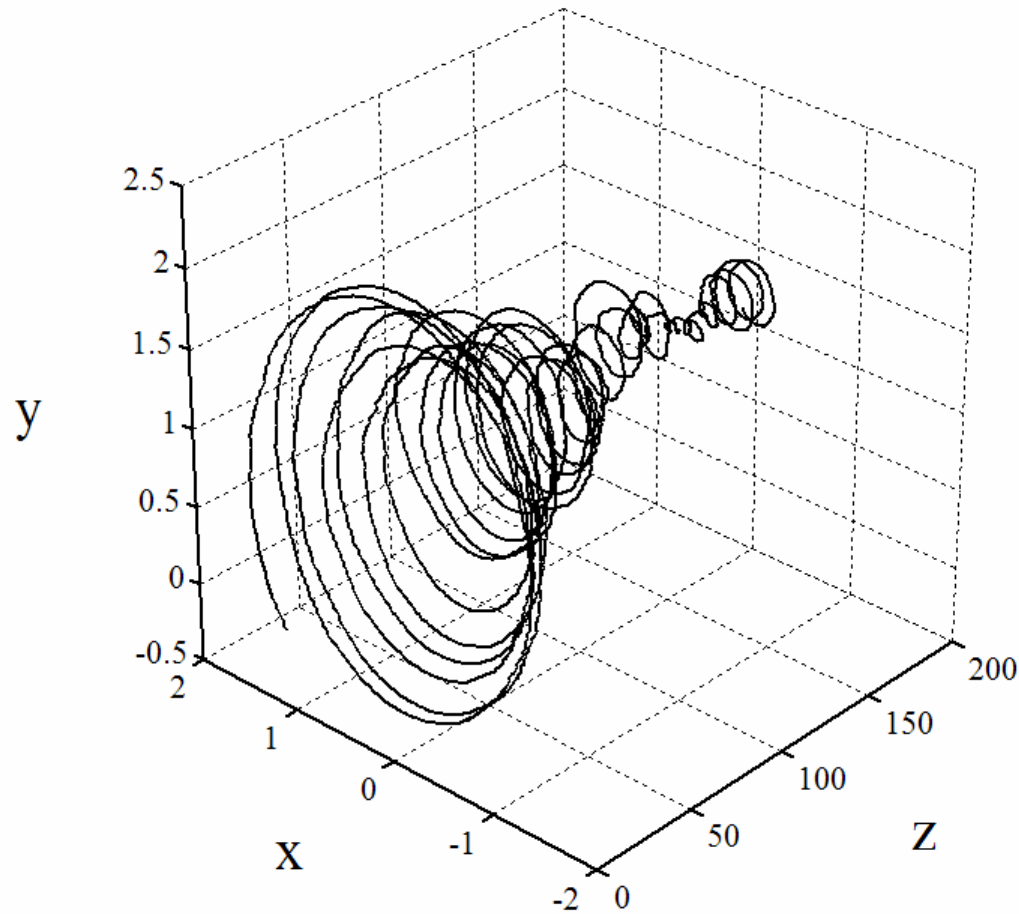


Übersicht über die analytischen Resultate in 0ter Ordnung

Classical	$\chi_{\parallel} = \frac{v_t^2}{2\nu} \quad \chi_{\perp} = \frac{v_t^2 \nu}{2\Omega_0^2}$	
Quasilinear	$D_{\mathcal{M}}^{(1)\nu=0} \sim \beta^2 \lambda_{\parallel}$ $\langle \delta x^2(t) \rangle_{\mathcal{M}}^{(1)} = 2 v_t D_{\mathcal{M}}^{(1)\nu=0} t$	$\lambda_{\perp} \rightarrow \infty, \nu = 0$ <p><i>No collisions.</i></p>
Subdiffusion	$D_{\mathcal{M}}^{(1)\nu=0} \sim \beta^2 \lambda_{\parallel}$ $\langle \delta x^2(t) \rangle_{\mathcal{M}}^{(1)} = 2 D_{\mathcal{M}}^{(1)\nu=0} \sqrt{\chi_{\parallel} t}$	$\lambda_{\perp} \rightarrow \infty$ <p><i>No perpendicular collisions.</i></p>
Rechester-Rosenbluth	$D_{\text{RR}}^{(2)} = \frac{16}{\sqrt{\pi}} \frac{\chi_{\parallel} \lambda_{\perp}^2}{L_K^2 \log^2 \left(\frac{32}{\sqrt{\pi}} \frac{\chi_{\parallel} \lambda_{\perp}^2}{\chi_{\perp} L_K^2} \right)}$	$L_K \approx \sqrt{\frac{2}{\pi}} \frac{\lambda_{\perp}^2}{4 \beta^2 \lambda_{\parallel}}$ <p><i>Collisions in all directions.</i></p>

*For better comparism with literature: $b_0 = 1$

Gestörte Trajektorie eines Teilchens (ALE)



Monte-Carlo-Code für Matlab7

prinzipiell zwei Verfahren:

- (a) Euler-Maruyama-Verfahren
- (b) **Milstein-Verfahren** (Runge-Kutta-Ansatz)

Ohne Beweis: beide Verfahren konvergieren.

Euler Vs. Lagrange

V-Langevin

- [1] H.D.Wang, R.Balescu et.al., Diffusive processes in a stochastic magnetic field, Phys.Rev.E 51(5) 4844 (1995)
- [2] E.Vanden-Eijnden, R.Balescu, Phys.Plasmas 3 (3) 874 (1996)

Fundamental

- [3] A.B.Rechester, M.N.Rosenbluth, Phys.Rev.40 38 (1978)
- [4] S.Corrin in Atmospheric diffusion and air pollution, edited by F.N.Frenkiel, P.A.Sheppard (Academic, New York,1990), p161

Percolation

- [5] B.B.Kadomtsev, O.P.Pogutse, Phys.Rev.Lett. 25 1155 (1970) & in Plasma Physics and Controlled Nuclear Fusion Research, Proceedings of the 7th International Conference, Innsbruck, 1978
- [6] M.B.Isichenko, Plasma Phys. Controlled fusion 33 795 (1991)
- [7] F.Casse, M.Lemoine, G.Pelletier, Phys.Rev.D 65 023002 (2002)

DCT

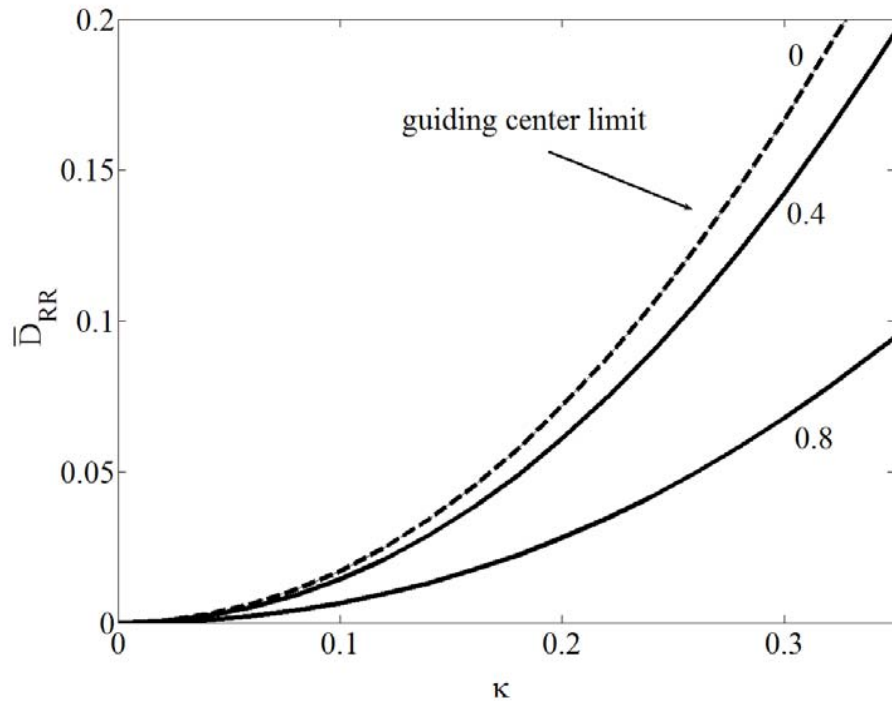
- [8] M.Vlad, R. Balescu et.al., Diffusion with intrinsic trapping in two-dimensional stochastic velocity fields, Phys.Rev.E 58(6) 7359

Flux tubes

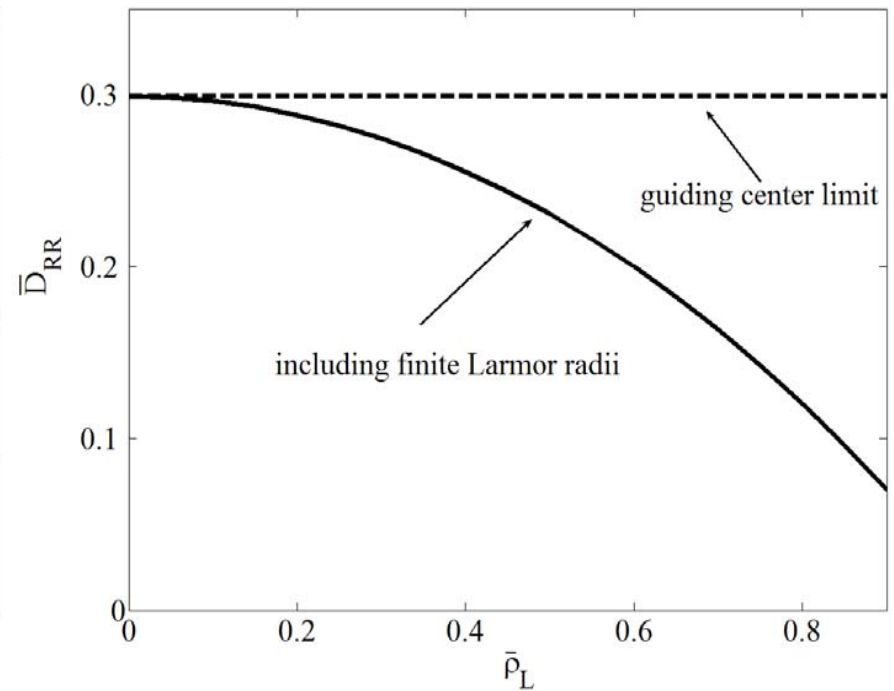
- [9] G.Qin, W.H.Matthaeus, J.W.Bieber, Transport in random magnetic fields: diffusion, subdiffusion and nonlinear second diffusion, Proceedings of the Tenth International Solar Wind Conference, edited by M.Velli, R.Bruno, F.Malara p664
- [10] W.H.Matthaeus et.al., Nonlinear collisionless perpendicular diffusion of charged particles, Astrophys. J. 590:L000, 2003

Proof of Isichenko

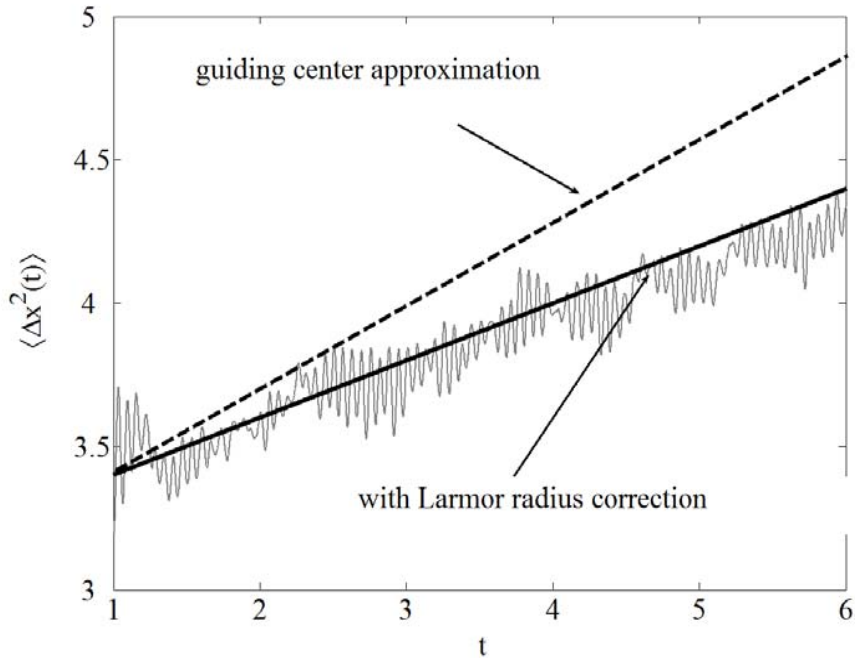
- [11] J.-D.Reuss, J.H.Misguich, Low frequency percolation scaling for particle diffusion in electrostatic turbulence, Phys.Rev.E 54(2) 1857 (1996)



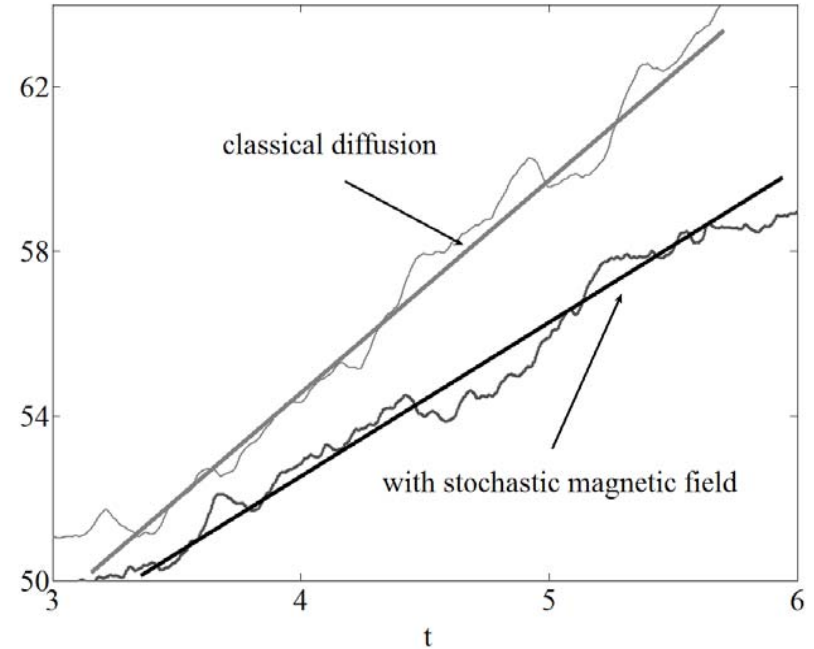
Influence of the Larmor radii on the diffusion in the Rechester-Rosenbluth regime. Total diffusion coefficient $\bar{D}_{RR} = \bar{D}_{RR}^{(0)} + \bar{D}_{RR}^{(1)}$ from the solution of the dimensionless Eqs. (3.42) and (3.48) for different values of $\bar{\rho}_L$. The classical diffusion rates were chosen in the typical Rechester-Rosenbluth range to be $\bar{\chi}_\perp = 0.4$ and $\bar{\chi}_\parallel = 20$.



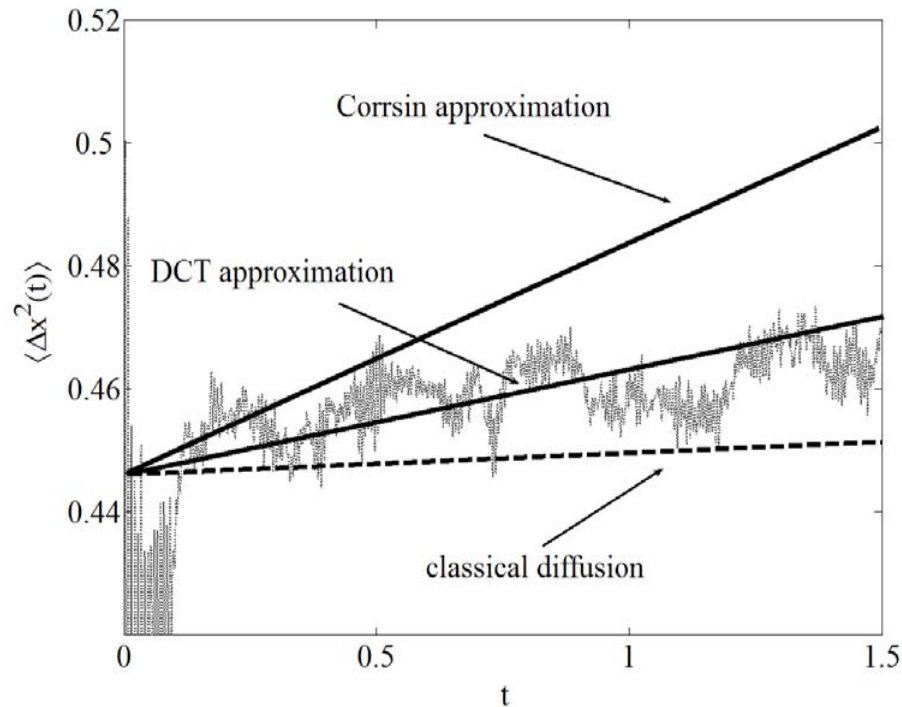
Diffusion coefficient as a function of the Larmor radius. Solution of the dimensionless Eqs. (3.42) and (3.48) for $\kappa = 0.4$, $\bar{\chi}_\perp = 0.2$ and $\bar{\chi}_\parallel = 20$.



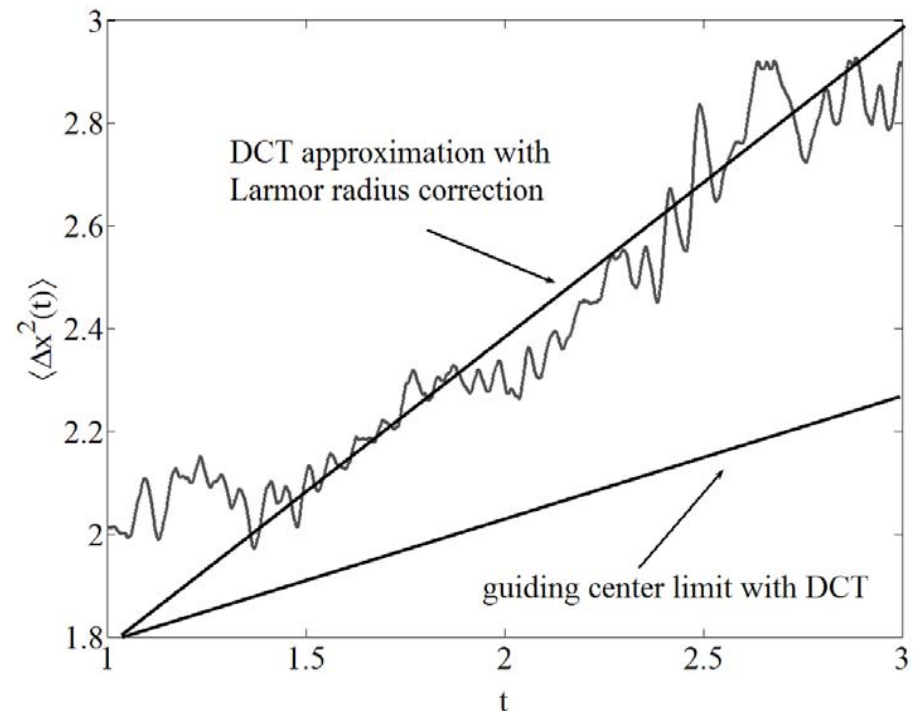
Numerical simulation of the A-Langevin equation and comparison with the analytical predictions, for the case of a strong guiding field. Shown is the MSD (in unit v_t^2/Ω^2) versus time (in units Ω^{-1}). The parameters are $\lambda_{||} = \lambda_{\perp} = 2 [v_t/\Omega]$, $\varepsilon = \beta/b_0 = 0.4$, $b_0 = 1.5$ and $\nu/\Omega = 0.05$.



Numerical simulation, with and without stochastic magnetic field, respectively, in the case of no guiding field. Straight lines indicate the analytical predictions. Shown is the MSD (in unit v_t^2/Ω^2) versus time (in unit Ω^{-1}). The parameters are $\lambda = 0.1 [v_t/\Omega]$, $\beta = 0$ and $\beta = 0.9$, respectively, $b_0 = 0$ and $\nu/\Omega = 0.2$.



Numerical simulation for high Kubo number (grey dots) and comparison of the DCT results and the Corrsin estimates. $b_0 = 5$, $\beta = 0.9$, $\lambda_{\parallel} = 365 \rho_L$, $\lambda_{\perp} = 0.85 \rho_L$, $\kappa = 438$ and $\nu/\Omega = 0.1$. The dashed line shows the classical diffusion coefficient.



A simulation for a high Kubo number and large values of ρ_L . Comparison of the DCT results and the guiding center estimates. $b_0 = 1$, $\beta = 0.5$, $\lambda_{\parallel} = 50 \rho_L$, $\lambda_{\perp} = 0.2 \rho_L$, $\kappa = 125$ and $\nu/\Omega = 0.2$. Straight lines indicate the analytical predictions for the DCT with Larmor radius corrections and the guiding center limit (also calculated with the DCT).

$$\Delta \approx 10^7 \rho_{L,cr}$$

- **Transport im Tokamak** (u.a. Verunreinigungen)

klassische Vorhersage: $\chi_{\perp} = \frac{v_t^2 \nu}{2 \Omega_L^2} \sim 10^{-4} \text{ m}^2 / \text{s}$ (Ionen)

experimenteller Befund: $D_{\perp} = \chi_{\perp} + D_{\perp}^{\text{AN}} \sim 1 \text{ m}^2 / \text{s}$

Anomaler Transport: Einfluß der **finiten Larmor Radien?**

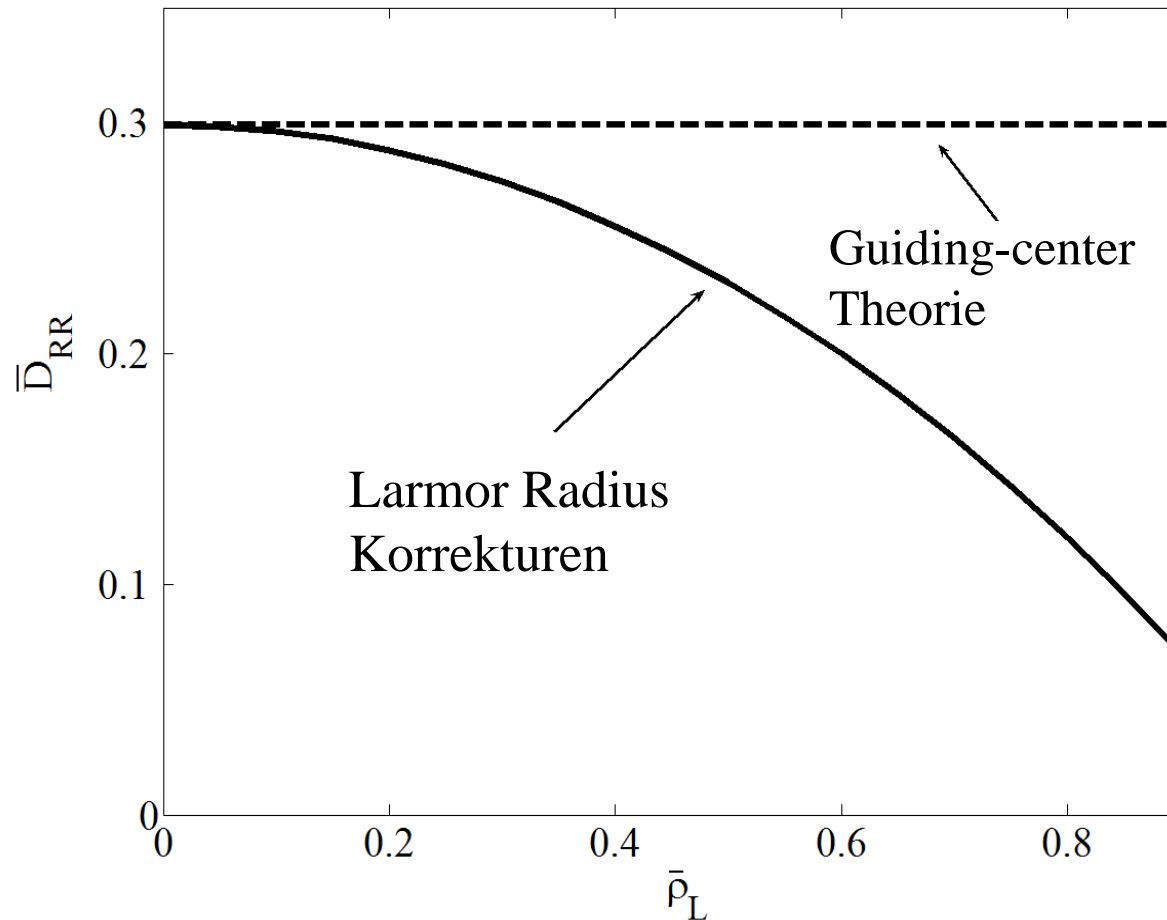
Mattioli et al. (ToreSupra), Galli et al. (JET)

- **Cosmic rays**

Larmor Radius: $\rho_{L,\text{cr}} \approx 1 \text{ AE}$

galaktische Magnetfelder: $B \sim 1 \mu\text{G}$

Einfluss des Larmor Radius am Beispiel des Rechester-Rosenbluth-Regime



Wie tragen magnetische Fluktuationen zum Transport bei?

- A-Langevin Modell für MSD und Diffusionskoeffizient

Welchen Einfluss hat der Larmor Radius auf die Diffusion?

- in Abhängigkeit der Kubo-Zahl : Reduktion oder Erhöhung

*Was passiert bei verschwindenden Führungsfeldern?**

- stochastische Felder wirken wie zusätzliche Stöße

Wie verändern Perkulationsstrukturen den Transport?

- Entstehung von Fluxtubes: Verringerung des Transports
- Endliche Larmor Radien: Erhöhung der Diffusion

Ausblick

- Berücksichtigung von stochastischen elektrischen Feldern